

Masterclass: Structure and Classification of C^* -algebras

Stuart White and Joachim Zacharias

4-17 Sep 2016

We will describe some of the recent progress in the structure and classification of simple separable unital and nuclear C^* -algebras which have recently led to the definitive classification theorem as well as its interplay with dynamical systems and coarse geometry. The first half of the course will develop aspects of the current state of the art, focusing on connections between simple nuclear C^* -algebras and injective von Neumann factors. The second half of the course will focus on finiteness of nuclear dimension in examples, particularly crossed products, through the Rokhlin dimension of dynamical systems.

Syllabus/Outline of Lectures

1. Overview. Factors, and type decomposition; Classification Theorems for amenable operator algebras; nuclear dimension.
2. Intertwining and classification: hyperfinite II_1 factor; AF C^* -algebras.
3. Structure of amenable operator algebras I. Ingredients in injectivity \Rightarrow hyperfiniteness
4. Structure of amenable operator algebras II. The Toms-Winter conjecture and tracial approximation.
5. Aspects of TAF classification
6. Nuclear dimension: various examples and techniques
7. Dynamical systems and crossed product; the idea of Rokhlin dimension
8. Nuclear dimension for crossed products by Rokhlin actions
9. Automatic finiteness of Rokhlin dimension, topological versions of the Rokhlin Lemma.
10. Dynamical asymptotic dimension and applications, further outlook.

References

Basics of operator algebras and K -theory

1. Ken Davidson. C^* -algebras by example. Fields Institute Monographs, 6. American Mathematical Society, Providence, RI, 1996.
2. Gerard Murphy. C^* -algebras and operator theory. Academic Press, Inc., Boston, MA, 1990.
3. Masamichi Takesaki. Theory of operator algebras. I. Reprint of the first (1979) edition. Encyclopaedia of Mathematical Sciences, 124. Operator Algebras and Non-commutative Geometry, 5. Springer-Verlag, Berlin, 2002.
4. Mikael Rørdam, Flemming Larsen and Niels Laustsen. An introduction to K -theory for C^* -algebras. London Mathematical Society Student Texts, 49. Cambridge University Press, Cambridge, 2000.
5. Bruce Blackadar. K -theory for operator algebras. Second edition. Mathematical Sciences Research Institute Publications, 5. Cambridge University Press, Cambridge, 1998.
6. Bruce Blackadar. Operator algebras. Theory of C^* -algebras and von Neumann algebras. Encyclopaedia of Mathematical Sciences, 122. Operator Algebras and Non-commutative Geometry, III. Springer-Verlag, Berlin, 2006.